Investigating the effectiveness of the least-angle strategy for wayfinding in unknown street networks

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Received 15 January 2004; in revised form 17 December 2004

Abstract. In this paper the wayfinding criterion always to 'proceed in the direction of the target', which is the core of the least-angle strategy, is examined. Although human route-selection behavior is much more complex than this, results of empirical studies presented in the literature suggest that preference for most direct direction plays an important role in human route choice within street networks. The least-angle strategy can be applied once the navigator knows the direction of the destination, and is therefore an appropriate temporary heuristic for situations where the navigator lacks more detailed information about the environment. To assess the effectiveness of the least-angle strategy in urban environments, an agent simulation has been performed on the urban street networks of two cities for three different modes of transportation—pedestrian, bicycle, and car. The results of the simulation are analyzed.

1 Introduction

Human wayfinding often takes place in unknown environments in which the navigator needs to make decisions under uncertainty. The wayfinder lacks the relevant knowledge required to select the optimal route, and must therefore rely on wayfinding heuristics. In this work a wayfinding behavior is described that may be applied in a situation in which the navigator
(a) lacks structural information regarding the environment (such as the position of landmarks, street names, the street hierarchy, and the direction of districts),
(b) cannot match information perceived from traffic signs with his or her destination (that is, the destination is not explicitly indicated on traffic signs and cannot be derived from perceived information),
(c) cannot ask other passersby, and
(d) has no external information sources such as maps or mobile navigation systems at hand.

Although this type of wayfinding situation seems to be somewhat artificial, it may occur temporarily within a more extensive wayfinding process. For these situations, the least-angle strategy (LA) may function as a useful temporary strategy until the next source of information (for example, a passerby, a 'you are here' map, or a useful traffic sign) is available. In such a situation, the information which the navigator possesses is the target vector (that is, the direction of the destination and its distance), and the direction of outgoing street segments from the current intersection (figure 1, over). On the basis of this information, the navigator tries to find a reasonable route among the options at hand.

The simulated navigator used in the computer simulations is given exactly the same knowledge, namely vectorial information concerning the destination and the direction of the streets leading from the current intersection. The simulated navigator obeys LA to find its way to the destination, and selects the route most in line with the target direction at each intersection. LA is a wayfinding heuristic that can be applied in cases in which the navigator wishes to find the shortest path (SP) but lacks structural knowledge about the environment. As with all wayfinding heuristics, LA may lead to
Figure 1. The navigator’s limited knowledge about the environment: the target vector and the directions of outgoing streets from his or her current position.

nonoptimal routes, that is, in this case it may not yield the SP to the destination. Through the use of LA, the navigator may fail to reach his or her destination at all, success being dependent on the network geometry. However, although wayfinding heuristics in general can lead to inaccuracies or asymmetries, they need to process only a limited amount of information in order to make a decision (Christenfeld, 1995); this requirement leads to faster decisionmaking than in the case of more complex strategies.

1.1 The test environments
As cities all over the world vary widely in their structure, it is impossible to find one single prototypical urban street network that can be used for the simulation. In this paper, a comparison is made between the results from the LA simulations of two European cities. To be specific, parts of the street networks from Vienna, Austria [figure 2(a)], and Bremen, Germany [figure 2(b)], are used for the simulation. Although the cities are taken from different countries, they are comparable in the structure of their respective street networks. The use of two cities rather than one city allows a better picture of the characterization of LA in terms of its effectiveness. Results may, however, be different for cities that have a different history and which evolved according to different planning rules. For example, the grid-like structure of typical US cities may yield a better performance with LA than the historic European cities with the oblique angles of their street network structures.

For the simulation three different types of navigator are distinguished, namely, pedestrians, cyclists, and car drivers. Cyclists and car drivers are required to obey one-way restrictions. Contrary to previous simulations of LA (Hochmair and Frank, 2002), in which, a 'standard' situation is used for the showcase in which the navigator reaches the goal, this work presents data from more-realistic situations which may cause the simulated navigator to fail to reach his or her destination.

The density of existent one-way restrictions varies substantially between the two cities (table 1). In the selected test areas, Vienna possesses a higher density of one-way restrictions than Bremen both for cars and cyclists. For all existing one-way streets for cars, Vienna provides a smaller percentage of bicycle lanes that permit cyclists to ride in the opposing direction (21%) than does Bremen (57%). Thus, in the Vienna street network, cyclists face a pattern of one-way restrictions that is much more similar to
Figure 2. The test environments: parts of (a) the Vienna street network and (b) the Bremen street network.

Table 1. Percentage of one-way streets in both test environments.

<table>
<thead>
<tr>
<th></th>
<th>Vienna</th>
<th>Bremen</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>bicycle</td>
<td>car</td>
</tr>
<tr>
<td>One-way streets (%)</td>
<td>55</td>
<td>69</td>
</tr>
<tr>
<td>One-way streets with two-way bicycle traffic (%) (a)</td>
<td>21</td>
<td>57</td>
</tr>
</tbody>
</table>

\(a\) That is, the percentage of the streets with one-way restrictions for cars which allow bicycle traffic in either direction.

that imposed on car drivers than is the case for the Bremen network. The impact of these differences in one-way restrictions on the effectiveness of LA will be investigated.

1.2 Objective of the work

Although there is evidence from observations of route-choice behavior that one common strategy is to proceed to the direction of the destination (Golledge, 1995; 1997), so far no attempts to operationalize this criterion can be found in the wayfinding literature. The contribution of this work to the wayfinding literature is to formalize LA and to test its effectiveness in urban street networks. In this paper LA is analyzed with respect to two characteristic values: the first value, which is called the success rate, is a measure of the reliability of the strategy. It describes the frequency with which a navigator who follows LA reaches the destination. The second value, which is called the performance factor, describes the detour caused by LA relative to the SP.

As many real street networks are highly complex and inhomogeneous, it would be extremely hard to find a mathematical model that can predict the success rate and the performance factor on the basis of network parameters alone. This is especially difficult in the case of LA where not only the network geometry but also the selected set of start-destination pairs influences the effectiveness of the strategy. Therefore the success rate and performance factor are derived by means of an empirical assessment of the simulation outcome rather than by computational geometry. Computational and
complexity issues which concern the implemented LA algorithm (that is, the memory
used and the computing time) are left aside, as LA is described as part of the human
wayfinding process and is not proposed for use in planning algorithms for other
domains, such as robot navigation. However, a formalization of the strategy will be
given to give the reader a better understanding of how the process works.

A navigator in a real-world situation may often have more information about the
environment than is actually needed to apply LA. This is the case when using a map,
following verbal route instructions, or acting in a familiar environment. In such
circumstances, the optimal route to the destination can often be found without the
use of heuristics, that is, the route can be determined with a high degree of certainty.
However, LA may function as an important ingredient in the mixture of strategies that
the navigator applies during a complete wayfinding process. It may be used especially
as a temporary strategy if more complex strategies cannot be applied, that is, if the
navigator lacks the knowledge required to apply such strategies. The following examples
provide motivating circumstances.

(a) A target description from a passerby may consist of a point in the direction and
rough distance information if a detailed route description is too complex for the
navigator to comprehend or the time is too short for a detailed route description.
(b) When navigating by car, if alone, the driver may have insufficient time to read the
detailed information that would be needed to memorize the exact route to the destina-
tion from a map. He or she must rely instead on the target direction gained from a
quick glance at the map.
(c) A traveler without a map may try to navigate to a distal landmark he or she perceives.
(d) A traveler may try to find a shortcut through an unknown district without navigation
aids.
(e) A traveler may get lost but maintain a sense of direction and consequently try to
find his or her way back to a known location.

1.3 Overview of the paper
The remainder of the paper is structured as follows: in section 2 a review is given
of previous findings concerning human route choice, and various simulation models of
route selection are presented. In section 3 LA is reviewed and formalized. Section 4
contains a description of the simulation setup. An analysis of the simulation results
with respect to the success rate is provided in section 5, which is followed by a
discussion of the performance factor in section 6. The paper concludes with section 7,
in which directions for future work are presented.

2 Human route selection behavior
2.1 Route selection criteria
Empirical studies have demonstrated a huge variety of route-selection criteria that
humans apply during the wayfinding process. Although shortest distance and least
time have been found to be the dominant choice criteria in various wayfinding studies
(Golledge, 1995; Stern and Leiser, 1987), people are not SP or least-time decision-
makers (Gärting et al, 1986; Golledge, 1997). This is especially true if the navigator
lacks spatial knowledge about the environment. In this case, he or she needs to rely on
small pieces of information and needs to apply wayfinding strategies which yield a route
that is at least close to the shortest or fastest route. Golledge (1997) claims that human
search space is sectoral, and that even an incorrectly perceived direction will make a
navigator eliminate turns that appear to direct him or her away from the approximate
target direction. A survey presented in the same paper provides evidence for the
importance of the 'general direction' of the destination in route choice. Although they
apply a variety of route-selection criteria, many people are not consciously aware of using specific wayfinding strategies (Golledge, 1999).

Hochmair and Karlsson (2005) investigated the importance of destination direction and initial route length for human route choice in unknown street environments with the use of a virtual desktop environment. Most of the decisions observed could be explained by LA, that is, by the participants' preference for the route segment leading in the general direction of the target, rather than by preference for the longest leg first, when they were asked to select the route that would lead them fastest to a distal target shown on the screen. Although wayfinding literature shows that preference for the SP is not the only criterion applied in human route-selection behavior, the navigator's preference for SPs is here taken as a norm. In situations in which this assumption is correct, LA serves as a simple but efficient temporary heuristic within a wayfinding process.

Various factors have been identified to explain the variety of criteria actually used in path selection. Stern and Leiser (1987) analyzed the correlation between spatial knowledge and route decisions. Without sufficient knowledge, the navigator settles into a pattern of stereotyped behavior and selects from a small number of known alternatives. Besides spatial knowledge, the trip purpose determines to a large extent which route attributes play a substantial role in route-choice behavior (Bovy and Stern, 1990). Scenery, for example, is very important for recreational trips, but irrelevant for work-related trips. The effect of exogenous variables, that is, uncertainty and time pressure, was analyzed in experiments by Stern (1999). The results revealed that navigators, when under time pressure, assign higher weight to salient features, apply noncompensatory decision rules more frequently, and are more willing to take risks. Golledge (1997) found that the structure of the environment (whether a grid, curves, or diagonal) as well as the orientation of the map affects the popularity of route-choice criteria. Asymmetry in route selection, that is, the taking of different routes to and from the same destination (given the same origin), can be explained by some underlying mechanisms, such as preference for the longest leg first (Bailenson et al, 2000; Christenfeld, 1995) or regionalization of the environment (Wiener and Mallot, 2003).

Discrete choice analysis (Ben-Akiva and Lerman, 1985) has played an important role in transportation modeling since the mid-1970s. From the categorization of navigators into different user classes or through the use of socioeconomic variables, the systematic heterogeneity (that is, the preference variation across individuals) of different user groups can be captured to some extent. The analyst has to identify the criteria (attributes) of each alternative that are likely to affect the choice of the individual. The analysis of choice behavior relies on assumptions about the decision rules which the navigator employs. These assumptions have been formalized within various models. Neoclassical economic theory introduces the concept of utility (Keeny and Raiffa, 1993), whereas the Luce model (Luce, 1959) and random utility models (Manski, 1977) are designed to capture uncertainty. Case studies which apply discrete choice analysis methods can, for example, be found in work by Khattak and Khattak (1998). The study uses a logit model to examine which independent variables (such as driver characteristics) influence navigators' spatial knowledge and en-route response to unexpected delay information. Han et al (2001) use a mixed logit model to investigate car drivers' choices including departure time and route. In particular, the study examines the impact of various traffic reports, each containing a description of an incident with different causes and levels of severity, on these choices.
2.2 Formal models on route selection

Graphs are a common abstraction to represent networks, such as street or routing networks. LA, applied to an abstract graph structure, uses the same choice mechanism as the Compass routing algorithm (Bose and Morin, 1999). The difference between the two algorithms is that LA aims to model human wayfinding behavior and thus may be extended with simulated effects of distortion due to imperfect perception or erroneous memorization (Hochmair and Frank, 2002). Both algorithms are local routing algorithms (Kranakis et al., 1999), which are also called online (Borodin and El-Yaniv, 1998) routing algorithms. What LA and the Compass routing algorithm have in common is that the navigator (and the network router which transfers the data packet, respectively) at no point in time has full knowledge of the network topology and geometry.

A routing algorithm is called memoryless if the next step taken depends only on the current position, the destination, and the outgoing edges from the current node. A geometric graph G defeats a routing algorithm A if there is a source-destination pair such that the navigator (or the data packet, respectively) never reaches the destination. The routing algorithm A is C-competitive for a class of graphs if the ratio of the path length resulting from A to the SP length for the same two nodes \( \leq C \).

Several works discuss the competitiveness and the success rate of the Compass algorithm and its variations for different types of artificial network. Bose and Morin (1999) show that the Compass algorithm is not C-competitive, that is, there exists no upper boundary for the detour caused by this algorithm. The authors also classify situations in which the moved packet gets trapped—never reaches its destination. However, the Compass algorithm always finds the destination within any Delaunay triangulation (Kranakis et al., 1999) and within any regular triangulations (Bose and Morin, 1999). Mitchell (2000) gives an overview of other online algorithms used for robot navigation, with a focus on the competitiveness of object-avoidance algorithms for various types of artificial networks.

Numerous simulations that were designed to assess the effectiveness of wayfinding strategies in partially unknown street networks are described in the wayfinding literature. Krek (2002) assesses the effects of incomplete information on the quality of the route through the simulation of a software navigator that seeks the SP in an existing urban street network. The navigator is provided with complete knowledge about the network geometry but has incomplete knowledge about one-way restrictions. Lóvás (1998) simulates wayfinding heuristics of differing complexity and storage requirements for evacuation from a maze. The results show that navigators with better memory can utilize more complex routing algorithms, which in turn result in an improvement of the wayfinding performance. Directional choice is also modeled in the paper. Duckham and Kulik (2003) empirically investigate the ratio of the route lengths found through the use of a simplest-path algorithm and an SP algorithm, in an application to an urban street network. As opposed to the previous approaches, these algorithms can use the complete information about the structure of the street network when the routes are computed.

3 Model formulation

3.1 The least-angle strategy

The following situation is envisioned: a traveler moves through an unfamiliar street environment and tries to navigate toward a distant destination, such as a church. Figure 3, which is adapted from figure 1, represents the situation. The navigator is located at an intersection P, from which the outgoing streets \( s_1 \) and \( s_2 \) can be perceived. It is also assumed that the navigator knows the direction to the destination D. He or she has no external map and no further knowledge about the network structure.
The angles between the outgoing streets and the destination direction are labeled \( a_1 \) and \( a_2 \). They are referred to as deviation angles. As the navigator directly perceives \( a_1 \) and \( a_2 \), and as he or she knows the direction to D, the magnitude of \( a_1 \) and \( a_2 \) can be judged. As \( a_1 < a_2 \) (that is, \( a_1 \) is more in-line than \( a_2 \) with the destination vector), the navigator chooses \( a_1 \) as the street by which to proceed when following LA.

A human navigator, when traveling through a street network, has several ways to derive the destination position. The destination may either be directly observed, or the navigator can use knowledge about his or her current position to maintain a sense of what the target vector is. Loomis et al (1993) classify the methods of updating position on the basis of kinematic order into position, velocity, and acceleration. In the case of position-based navigation, external signals from visible, audible, or odorous landmarks indicate the traveler's position. Without external landmarks, the traveler's navigation relies on external or internal signals which indicate course and speed. If the emphasis is on velocity signals that are produced by self movement, the process is called path integration or dead reckoning. If the emphasis is on acceleration signals, the process is called inertial dead reckoning. To determine the angular displacement since the last known heading, the traveler needs to integrate the turn rate over time. In this simulation, the destination in the traveler's mental map is defined as the vector between his or her current position and the destination. During travel, the traveler updates his or her mental position continuously with respect to the origin, which yields an update of the target vector. The method by which one's destination is defined from a target vector is called vector encoding (McNaughton et al, 1991).

As the view of the destination or other landmarks may be blocked by other buildings when an urban street network is being navigated, a human navigator will have to rely on path integration when obeying LA. In such cases the navigator's believed destination position may be distorted by path-integration errors (Klatzky et al, 1997; Loomis et al, 1993). Perception errors (Downs and Stea, 1973; Loomis and Klatzky, 1999; Sadalla and Montello, 1989) and encoding errors (Huttenlocher et al, 1991) may cause additional distortion of the perceived destination position. However, LA is here considered as a temporary strategy that will be used for only a small number of sequential turns. Once the navigator can perceive additional information from the environment again, LA can be followed by a more precise and sophisticated strategy. Therefore, as the average number of turns is expected to be small, potential effects caused by path-integration, perception, and encoding errors (Hochmair and Frank, 2002) are left aside and an error-free navigator is used in the simulation. As LA is considered as a temporary strategy, the distance between the agent's initial position and destination, as the crow flies, is restricted in the simulation to 500 m (section 4).
3.2 The memorized least-angle strategy
Whereas LA is memoryless, a slightly modified version, called the memorized least-angle strategy (LA'), is actually used for the simulation. For the description of LA', the term agent—a concept from artificial intelligence—is used as a conceptual paradigm to represent the human navigator. This allows an elaboration of the problem on a more abstract and theoretical level, and reduces the complexity of human navigation. The agent governed by LA' uses a local memory that memorizes the last node visited. With LA', the agent behaves in exactly the same way as with LA, with the one difference that the agent is not allowed to return to the previous node in its subsequent step (dead ends are an exception). This restriction prevents oscillation back and forth along the same section of a road, which would otherwise frequently occur within the test networks that are used for the simulation. Such oscillation would yield a simulated navigation behavior very different to human route-choice behavior.

Figure 4 illustrates the difference between LA and LA' through a comparison of the different outcomes of a specific decision situation given in the Vienna street network. Let us assume that the simulated agent uses a car, so that one-way restrictions need to be obeyed. The agent starts its trip at node 149 and attempts to reach destination D. After traversing node 151, the agent reaches node 8157. a, < a, at node 8157 and therefore the agent moves on to node 8139 both in the case of LA and in the case of LA'. At node 8139, $\beta_1 < \beta_2 < \beta_3$, thus, the agent will return to node 8157 if dictated by LA, followed again by a move to 8139, and so on. In this way, the agent will oscillate forth and back between 8157 and 8139 [see figure 4(a)] and never reach its destination.

Contrary to this behaviour, an agent that obeys LA' is not allowed to backtrack from node 8139 to node 8157. Instead, the agent selects from one of the remaining alternatives, which are the street segments leading to node 150 and 8144 [figure 4(b)]. As $\beta_2 < \beta_3$, the agent continues its trip to 8144, from where it selects the alternative which is most in line with the target vector (that is, the agent makes a left turn). Finally it reaches its destination after traversing two more decisions points.

As LA' applies a heuristic that is closer to actual human wayfinding behavior than LA, further descriptions, such as formalization, or the investigation of success rate and performance factor, are restricted to LA' and LA is left aside.

![Figure 4](image_url)
3.3 Formalization

In this section LA' is formalized in pseudo code. A demonstration is given of how the algorithm that implements LA' is structured. The implementation of LA' was performed in Haskell, an executable, functional programming language (Thompson, 1999). As the agent acts on a street environment that is abstracted as a graph, the concepts of graph theory that are used for the explanation of the LA' formalization are reviewed.

A graph, \( G = (V, E) \), comprises a set of vertices \( V \) and edges \( E \) which connect these vertices. Each edge is described by the pair of vertices (called endnodes) that it links. If two nodes have a common edge the nodes are said to be adjacent. An edge is said to be incident with a node if that node is one of its endnodes. A loop is an edge whose endnodes are identical. A graph that contains no loops or parallel edges is called a simple graph. The number of distinct edges incident with a node is called the degree of the node.

An undirected graph [figure 5(a)] is a graph whose edges are unordered pairs of vertices. A directed graph has directions assigned to its edges [figure 5(b)] and the edges are represented as arrows. In a directed graph, the outdegree, \( d^+ \), of a node is the number of edges leaving the node, the indegree, \( d^- \), of a node is the number of incoming edges incident with that node. A graph (directed or undirected) is connected if there is a possible path between any two nodes.

Figure 5. (a) An undirected graph and (b) a directed graph. Nodes and edges are numbered, with edge labels denoted by the prefix \( e \).

A walk through a graph is a sequence of nodes \( (V_1, V_2, \ldots, V_n) \) for which any two adjacent nodes \( V_i \) and \( V_{i+1} \) are the endpoints of some edge. If the edges in a walk are all distinct it is called a trail. A trail is called closed if \( V_n = V_1 \), otherwise it is open. A trail is called trivial if it consists only of a single vertex; otherwise it is nontrivial. A nontrivial closed trail is called a circuit. An example of a circuit [in figure 5(b)] is the sequence \( (4, 3, 5, 4) \). A circuit \( (V_1, V_2, \ldots, V_n, V_1) \) \((n \geq 3)\) whose \( n \) vertices are distinct is called a cycle. An example is the sequence \( (4, 3, 5, 4) \). For further definitions of basic concepts in graph theory the reader is referred to any introductory textbook (for example, Chartrand and Lesniak, 1986; Piff, 1991).

The LA' algorithm is based on a repetitive sequence that consists of perceive, decide, and move operations. It therefore follows the decomposition of an agent's control system into three functional elements (Nilsson, 1980), namely sense, plan, and act.

The repetitive cycle of the LA' algorithm is structured as follows. As long as the agent's position does not match the target, the agent perceives the set of edges that are incident with its current position. Within the decision process, the agent filters those edges that it is allowed to traverse and that do not lead back to the previous node. Thus, the agent is required to remember its previous node. The filtering of legal routes is achieved by a cost function, \( c \), which assigns to an edge an indefinite weight if the agent is not allowed to traverse that edge from its current position. If the set of filtered
edges is nonempty, the agent computes the deviation angle $a$ for each edge from the set, and selects the street segment with the smallest $a$. Otherwise, the edge that leads back to the previous node is selected. Such an edge must exist as the graph is assumed to be connected. After the decisionmaking process the agent updates its position, the previous node, and the target vector. The next cycle then begins.

LA' algorithm

Initial conditions: $G = (V, E)$ is a simple, directed, and connected graph: $s \in V$ is the starting vertex, $t \in V$ is the target vertex, $p \in V$ is the vertex of the current position, $f \in V$ is the previously visited ('former') node. $c(e_{i,j}): E \rightarrow \mathbb{R}^+$ are the costs of the edge $e$ between vertices $i$ and $j$. $e_{i,j}$ converts an edge into a vector from $i$ to $j$. $v_{i,j}: V \times V \rightarrow V$ is the polar vector between vertices $i$ and $j$. $d \in V$ is the target vector. $i \in \mathbb{R}^+$ stores the traveled path length. $a(e, d): E \times V \rightarrow \mathbb{R}$ is the deviation angle for edge $e$. $e(i) = \{e_{i,j} \subseteq E\}$ is the set of edges that are incident to $i$. $\text{pos} \in \mathbb{R} \times \mathbb{R}$ is the coordinate vector of the agent's current position.

```plaintext
p \leftarrow s
\text{pos} \leftarrow (0, 0)
f \leftarrow p
l \leftarrow 0
d \leftarrow v_{o,i}

\textbf{while } p \neq t
\begin{align*}
\varepsilon(p) &\leftarrow \{e_{i,j} \in E : i = p\} \quad \text{ perceive} \\
\varepsilon'(p) &\leftarrow \{e_{j,i} \in \varepsilon(p) : c(e) \neq \infty \wedge j \neq f\} \quad \text{ decide} \\
\text{if } \varepsilon'(p) \neq \{\} \text{ then} \\
\quad e_{p,q} &\leftarrow \argmin_{e \in \varepsilon'(p)} a(e, d) \\
\text{else} \\
\quad e_{p,q} &\leftarrow e_{p,l} \\
\quad p &\leftarrow q \\
\text{move} \\
\quad \text{pos} &\leftarrow \text{pos} + e_{p,q} \\
\quad p &\leftarrow q \\
\quad d &\leftarrow d - e_{p,q} \\
\quad l &\leftarrow l + |e_{p,q}|
\end{align*}
\textbf{endwhile}

\text{return } l
```

4 Setup of the simulation

The network graphs used in the simulation (figure 2) contain, besides the network geometry, information concerning one-way restrictions, bicycle lanes along streets, and trespassing restrictions for pedestrians, cyclists, and cars. Both graphs are abstractions of the urban street environments, and they omit indoor and restricted areas (such as a hospital campus). As a consequence of driving restrictions (one-way streets, for example), and built obstacles (roads blocked to cars, etc), the route found between two nodes can vary between car, bicycle, and pedestrian navigation modes. The variation of restrictions for a single street segment between two nodes $A$ and $B$ yields a total

<table>
<thead>
<tr>
<th>A - B</th>
<th>B - A</th>
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<tbody>
<tr>
<td>Pedestrian</td>
<td>Y/N</td>
</tr>
<tr>
<td>Bicycle</td>
<td>Y/N</td>
</tr>
<tr>
<td>Car</td>
<td>Y/N</td>
</tr>
</tbody>
</table>
of $2^6 = 64$ combinations for the three navigation modes (table 2). In the permission matrix, Y means that the street segment can be traversed in the given direction and by the transportation mode described, whereas N indicates that such action is physically impossible or is not allowed.

The theoretical number of sixty-four combinations is reduced as a result of several correlations between the restrictions found in the actual street data of the test environments. First, the permission for pedestrian navigation is symmetric, as there are no one-way restrictions for pedestrians (eliminates thirty-two combinations). Second, cycling restrictions are symmetric if car traffic is completely forbidden (eliminates four combinations from the remaining set). Third, cycling restrictions are also symmetric if car traffic can enter a street from both sides (reduces the set by four combinations). Fourth, it is never found that the only possible driving direction for car and bicycle mode on the same street segment are in opposite directions (a further reduction of four combinations). Fifth, pedestrian navigation is possible if cyclist navigation is possible at least in one direction (eliminates six combinations). Sixth, bicycle navigation is possible in at least one direction, if car navigation and pedestrian navigation are allowed in any direction (three combinations). Seventh, no route is included that cannot be traversed with any of the three transportation modes in at least one direction (removes one combination). Ten restriction combinations for the three transportation modes remain (table 3).

Table 3. Restriction combinations for traversing a street as found in both test networks where the permission matrices shown follow the format of table 2. $\neg c$, $\neg b$, and $\neg p$ symbols indicate that a street segment cannot be traversed by a car, a bicycle, or a pedestrian, respectively. Arrows denote one-way restrictions and an accompanying b indicates the presence of a bicycle lane which allows a cyclist to travel in either direction.

<table>
<thead>
<tr>
<th>(a)</th>
<th>(b)</th>
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<tbody>
<tr>
<td><img src="image1" alt="Diagram of street network combination A-B" /></td>
<td><img src="image2" alt="Diagram of street network combination A-\neg p-b-B" /></td>
</tr>
<tr>
<td>(c)</td>
<td>(d)</td>
</tr>
<tr>
<td><img src="image3" alt="Diagram of street network combination A-\neg b-c-B" /></td>
<td><img src="image4" alt="Diagram of street network combination A-\neg c-B" /></td>
</tr>
<tr>
<td>(e)</td>
<td>(f)</td>
</tr>
<tr>
<td><img src="image5" alt="Diagram of street network combination A-b-B" /></td>
<td><img src="image6" alt="Diagram of street network combination A-B" /></td>
</tr>
<tr>
<td>(g)</td>
<td>(h)</td>
</tr>
<tr>
<td><img src="image7" alt="Diagram of street network combination A-\neg p-b-B" /></td>
<td><img src="image8" alt="Diagram of street network combination A-\neg p-b-B" /></td>
</tr>
</tbody>
</table>

The simulation procedure is as follows: twelve nodes spread over each network were selected as destinations. For each of the destinations, all nodes that are located within a radius of 500 m from the destination were used as start points (figure 6, see over). For all start-destination pairs, both the SP and the routes found with LA' were computed for each of the three transportation modes. The number of used start-destination pairs amounts to over 600 for each transportation mode in each environment.
5 Reaching the target: assessing the success rate
The success rate of a strategy is defined as the ratio of the number of trials for which the destination was reached to the total number of trials. Thus, the higher the success rate of a strategy, the more reliable is the strategy with respect to arrival at the destination. However, the success rate does not provide information about detours that may be caused by the strategy which is applied (this will be discussed in section 6). This section presents the first part of the simulation results and compares the LA' success rates obtained for the two street networks and the three transportation modes, respectively.

5.1 Results
In figure 7 the success rates of LA' are presented for all transportation modes in the Vienna and Bremen street networks. The interurban comparison of success rates for each transportation mode reveals relatively small differences. The Vienna network displays a significantly better success rate for pedestrian navigation (Vienna: 99%, Bremen: 97%; Mann-Whitney: \( n_{\text{Vienna}} = 834, n_{\text{Bremen}} = 621, p = 0.00 \)), whereas Bremen has a significantly better success rate for cyclists (Vienna: 83%, Bremen: 90%, \( p < 0.02 \)).
and a nonsignificantly better success rate for car drivers (Vienna: 83%, Bremen: 87%, p > 0.39). The better success rate for Bremen may be explained by the smaller fraction of one-way restrictions for cyclists and car drivers in Bremen compared with Vienna (compare table 1). Thus, in the Bremen network cyclists and car drivers can move 'more directly' to their destination than in Vienna. On average, over all transportation modes, Bremen has a nonsignificantly better success rate than Vienna (Vienna: 88%, Bremen: 91%, p > 0.18).

One clear tendency can be found from an intraurban comparison of success rates between the three transportation modes (table 4). In both cities, the simulated agent reaches its goal significantly more often in pedestrian mode than in bicycle or car mode. This finding represents evidence that one-way restrictions reduce the chance to reach one's destination when under the direction of LA'.

Table 4. Significance of the differences between success rates for pedestrian, bicycle, and car transportation modes (Mann–Whitney test).

<table>
<thead>
<tr>
<th></th>
<th>Vienna</th>
<th>Bremen</th>
<th>a (two-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedestrian</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Bicycle</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Pedestrian</td>
<td>0.035</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Bicycle</td>
<td>0.035</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

5.2 Cycles and circuits

Urban street networks can be abstracted as graphs. As shown previously (section 5.1), the two graphs defeat LA'. That is, the agent may get caught in a circuit or cycle and may therefore never reach its destination.

For the analysis of the success rate, it is of interest to have a criterion that decides whether the agent is caught in a cycle (or circuit) or not. Once it is found that the agent is caught in a cycle (or circuit), the navigation simulation for the corresponding start–destination pair can be interrupted and begun for the next pair. To be caught in a circuit, the agent must enter any vertex \( V_i \) of the street graph at least twice from the same previous node \( V_j \) with LA' the only parameter which affects the decision outcome at a revisited node \( V_j \). All other parameters, namely the target vector between \( V_j \) and the destination (as the agent is simulated to be error free), and the network geometry, stay constant between recurrent visits. This means that a node may be visited several times (from different directions) within a circuit. If the edge from which the agent enters each revisited node is always the same, the agent is caught in a cycle, as no node is visited twice in the interim.

Figure 8 (see over) shows two situations taken from the simulation in which the agent does not reach its destination. Let us assume that the agent needs to obey one-way restrictions. Figure 8(a) displays an example where the agent is caught in a cycle. The agent starts at 8035 and heads towards destination D. After moving to 8056, the agent becomes caught in the recurrent node sequence (8056, 8071, 8002, 8055, 8056). After the agent has entered node 8071 for the second time through the same edge (that is, from 8056), the criterion which tells us that the agent will never reach its destination is satisfied. Analysis of the recurrent sequence of visited nodes (8071, 8002, 8055, 8056, 8071) shows that each node is visited only once, that is, the agent is caught in a cycle (and not in a circuit).

Analysis of the recurrent sequence of revisited nodes (366, 363, 486, 364, 486, 363, 366) in figure 8(b) shows that at least one node within this sequence is visited twice
Figure 8. The memorized least-angle strategy: the agent is caught in (a) a cycle and (b) a circuit.

(specifically, 363, and 486), because these nodes are entered from different directions. Therefore, contrary to figure 8(a) the agent gets caught in a circuit and not in a cycle.

A more intelligent algorithm that is possibly a closer approximation to human route choice as concerns the repeated traverse of edges, is the *memorized random choice* algorithm (Lovás, 1998). This algorithm remembers which of the edges the navigator has traversed before, and selects the edge among the set of alternatives that has been previously traversed least often. Integrating such heuristics into LA' would help to overcome the problem of becoming caught in a cycle or circuit.

6 Detours: assessing the performance factor

The performance factor provides a measure of the detour of a path found by LA' relative to the SP between the same pair of start–destination nodes. The performance factor is expressed as the ratio of the LA' path length to the SP length, which yields a value greater than or equal to one.

6.1 Visualization of the results

Figure 9 contains scatter plots, in which the performance factors of LA' paths are plotted against SP lengths for all start–destination pairs. The six plots display the results for each transportation mode in both street networks. The points with zero factor values correspond to start–destination pairs for which the agent failed to reach the destination. The various numbers of dots with zero values in the case of each transportation mode within each city are a manifestation of the finding with regard to the success rates (section 5.1), namely that LA' yields a higher success rate in pedestrian mode [figures 9(a) and 9(d)] than in cyclist or car transportation modes.

6.2 Intraurban comparison of the performance factors

The intraurban differences in the performance factor between pedestrian, bicycle, and car transportation modes vary between both cities (see table 5, and figure 10). Whereas the Vienna street network yields a similar performance factor for bicycle and car transportation modes (bicycle: 1.11; car: 1.13), a similar feature can be observed in the case of pedestrian and bicycle transportation modes in Bremen (pedestrian: 1.10; bicycle: 1.10). In other words, the average detours faced by cyclists and car drivers in Vienna that obey LA' are about the same. This finding may be explained by the similar amount of one-way restrictions both for car drivers and cyclists in the selected part of the Vienna street network (compare table 1). The fact that only 26% of the streets in the Bremen test network have one-way restrictions for cyclists (compare table 1) may explain the similar performance factors for cyclists and pedestrians in the navigation of this city. The results nicely demonstrate the impact of the legal restrictions in street networks on the performance of LA'.
Figure 9. The performance factors of LA' for all transportation modes in both of the street networks: (a) pedestrian (Vienna), (b) bicycle (Vienna), (c) car (Vienna), (d) pedestrian (Bremen), (e) bicycle (Bremen), and (f) car (Bremen).

Table 5. The performance factors of LA' for all transportation modes in both of the test networks. → denotes a significant difference of average performance factors (p < 0.05) between two transportation modes or two cities respectively.

<table>
<thead>
<tr>
<th></th>
<th>Average pedestrian</th>
<th>Average bicycle</th>
<th>Average car</th>
<th>Average over all modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vienna</td>
<td>1.06</td>
<td>1.11</td>
<td>1.13</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
<td>↑</td>
</tr>
<tr>
<td>Bremen</td>
<td>1.10</td>
<td>1.10</td>
<td>1.26</td>
<td>1.15</td>
</tr>
</tbody>
</table>

Figure 10. The average performance factors of LA' for the Vienna (crosshatched) and Bremen (striped) street networks.
6.3 Interurban comparison of the performance factors

A comparison of the performance factors computed for both cities shows that for all transportation modes, except for bicycle, the Vienna network provides a better (that is, a smaller) average performance factor than that provided by the Bremen network (table 5 and figure 10). In consideration of the different densities of one-way streets found in the two street networks, one would expect the opposite result, namely a better performance factor in the case of Bremen. A possible explanation for the somewhat worse performance factors of the Bremen network, however, is the large number of streets in this network that have separate lanes for each direction. The lanes are separated by physical barriers or by tracks of the tramway, which prohibit both the crossing of the streets and also left turns at numerous intersections. These restrictions affect car transportation especially. Thus, a lane that is entered in one direction can lead far from the destination direction before the next intersection is reached that allows a left turn or a U-turn to be made. That is, one particular decision (namely, to enter a separated lane with turn restrictions), may lead to a long total path. From figure 9(f) it can be seen that the maximum performance factor for car transportation in the Bremen network is 7.1 (in the case of Vienna it is 4.0). This long detour is caused by a route that runs along a street with separate lanes for each direction.

To provide a better picture of the performance factor distribution besides average values, the percentage of successful LA' trials which yield a path no more than 10% (and, respectively, 50%) longer than the corresponding SP was computed in each mode. The results (table 6) show larger values for the Vienna network than for the Bremen network in each of the categories, which means that LA' provides a higher percentage of 'short' paths in the case of Vienna than it does in the case of Bremen. In the Vienna network, for example, 77% of routes found for the car transportation mode with LA' are no more than 10% longer than the SP, whereas for Bremen the corresponding value amounts to only 70%. These results are in good agreement with the findings as regards the average performance factors in both cities (table 5), where the Vienna test network has a better performance. In summary of the values in table 6, it can be said that LA' yields a route of reasonable length for most start–destination pairs in both of the cities and for all transportation modes.

Table 6. The distributions of path lengths computed with the LA' algorithm in the Vienna and Bremen street networks.

<table>
<thead>
<tr>
<th></th>
<th>Vienna pedestrian</th>
<th>Vienna bicycle</th>
<th>Vienna car</th>
<th>Bremen pedestrian</th>
<th>Bremen bicycle</th>
<th>Bremen car</th>
</tr>
</thead>
<tbody>
<tr>
<td>% of routes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt; 1.5 x SP length</td>
<td>98</td>
<td>95</td>
<td>93</td>
<td>93</td>
<td>94</td>
<td>83</td>
</tr>
<tr>
<td>&lt; 1.1 x SP length</td>
<td>88</td>
<td>81</td>
<td>77</td>
<td>76</td>
<td>75</td>
<td>70</td>
</tr>
</tbody>
</table>

7 Conclusions

7.1 Summary

This work contained a discussion of the results from a wayfinding simulation that was operated on parts of the street networks of two different cities (Vienna and Bremen) with the use of three transportation modes (pedestrian, bicycle, and car). The objective of the simulation was to investigate the effectiveness of the memorized least-angle strategy (LA') when applied in urban environments. With LA', the simulated agent always selects the street segment most in line with a vector towards the target, and avoids backtracking to its previous position. Simulation results were analyzed with respect to the success rate and the performance factor.
An intraurban comparison of results for the three transportation modes reveals that one-way restrictions decrease the navigator’s chance of reaching the destination when following this strategy. One-way restrictions also yield longer detours. As pedestrians do not need to obey one-way restrictions, LA’ was found to be most effective in terms of success rate and performance factor for the pedestrian transportation mode. An interurbann comparison of computed routes suggests that streets with separated lanes for both driving directions diminish the performance rate and cause longer detours. The results for success rate and performance factor show that even a navigator who has very limited knowledge about the environment can in most cases find his or her destination on a path of reasonable length when obeying LA’.

Because of the perception errors and memory errors of human navigators, LA’ is in real networks limited to a maximum path length and a maximum number of turns. Otherwise the cumulative errors in the navigator’s mental update of the position will lead to strong distortions in the perceived target vector and, consequently, to incorrect decisions. Therefore, LA’ can be used as a temporary strategy but will fail as a long-term strategy, at least if the target vector cannot be assessed from time to time. However, one of the strengths of LA’ is that it requires little cognitive effort for the decisionmaking process. The decisionmaker takes into account only the directions of the streets at the current intersection, and the destination direction. Therefore, LA’ can be used for situations in which decisions have to be made quickly, and where no detailed information is available at hand.

7.2 Future work
Heading in the general direction of the destination is an important part of human route-choice behavior. Various strategies may be applied to achieve this task. That is, at a given intersection, the decisionmaker may sometimes prefer not to select the street segment that is most in line with the destination direction. Thus, LA’ as presented in this paper is highly abstracted from actual human route-choice behavior. Even under the assumption that the navigator has only slightly more knowledge about the environment than the agent was given in the simulation, additional factors will be taken into account by a human wayfinder in route selection. Besides deviation angle, the following parameters (which can all be perceived at a decision point) may among others play a role in the navigator’s choice between street alternatives:
(a) the length of the initial street segment,
(b) the number of streets intersecting with an initial street segment,
(c) the hierarchy or width of the initial street segment,
(d) the regionalization of the environment.

Future work should use human-subject testing to assess the importance of these criteria for the human route-choice behavior. Also, if vector-based wayfinding strategies are to be considered in indoor environments the list of parameters will need to be extended. Once the relevance of further parameters for the navigator’s preference behavior is known, the decision models may be merged with strategies already included in existing prototypes of navigation simulators. Important fields of navigation simulation are, for example, emergency evacuation (Lővás, 1998), pedestrian-flow and passenger-flow modeling (Hoogendoorn and Bovy, 2004), and the planning of signage in airports (Raubal, 2001).

Adequate accuracy of the mental target-vector update is the basis for the application of vector-based navigation strategies if the target cannot be permanently perceived. The mental updating process is primarily based on human path-integration abilities, which have been thoroughly tested in laboratory environments (Klatzky et al, 1997; Loomis et al, 1993; Loomis and Klatzky, 1999; Mallot et al, 2002).
However, empirical studies about spatial orientation and path-integration abilities in urban environments are only sparsely found in the wayfinding literature (Montello, 1991). Thus, human-subject testing in street environments would help to investigate the limits of vector-based navigation in urban environments, and would help to assess the magnitude of angular and distance distortions that cumulate in the navigator’s cognitive map during the wayfinding process. This, in turn, would allow the extension of existing wayfinding models, such as LA', through the addition of an error term to obtain more realistic results.

Acknowledgements. I would like to thank Alenka Krek for providing me with the data of the Vienna street network, and Jan-Oliver Wallgrin and Jianhui Liu for their valuable comments on the formalization section. Elizabeth Leppman helped to improve the grammar and style of this work. This research has been funded by the IQN grant #40300059 from the German Academic Exchange Service (DAAD).

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